# Secant Line

Dr. Reinhard Simonovits, Univ. Graz

Wenn sie fragen zum Projekt *M@th Desktop* haben oder beta testen wollen, bitte kontaktieren Sie

Dr. Reinhard Simonovits,
Reinhard.Simonovits@uni-graz.at

Diese Lerneinheit beschreibt das Sekanten Problem.
Es ist eine Lehrerversion, da alle Beispiele durchgerechnet sind.
Aus Gründen der Länge für den Ausdruck sind nicht alle Kapitel offen.
Die zugehörige Palette ist nicht ausgedruckt.

## 1 THE BALLOON MOVIE : Flying a balloon

### 1.1 Blowing up a Balloon

For a competition a balloon like a sphere is filled with hot-air.
As you change the volume V of air, the radius r changes.

The formula reads $r[V] = \sqrt[3]{\frac{4}{3\pi} V}$ `(*  r in inch, V in yd³ *)`

It seems that the balloon blows up faster at the start and then slows down as you blow more air into it.
What you notice is the rate of change of the radius $\Delta r$ with respect to change in volume $\Delta V$.

### 1.2 The Balloon Movie

**Let's experiment a little**... Try different **$\Delta V$'s** between 5 $yd^3$ and 50 $yd^3$.

Keep your eye on the plot label at the top of the graph. Note the change in value of the average rate of change of the radius with respect of change in volume as you *increase* or *decrease* the value of $\Delta V$.

As the balloon grows, the same small change in volume produces a much smaller change in the radius.

Press the START button to produce the movie.

```
ΔV = 60.0; (* in yd³ *)
MDBalloonMovie[ΔV]
```

Start ▷ | Stop | Delete × | Print

## 1.3 Questions to the Movie

In order to answer the questions you need a printout of the movie.
1. START the movie
2. Go to the menu–barto *M@th Desktop / Enter Name for Printout.*
3. Print out the movie.

```
ΔV = 30.0; (* in yd³ *)
MDBalloonMovie[ΔV]
```

Start ▷ | Stop | Delete × | Print

Take your movie print with $V_0 = 90\ yd^3$ and work through the following problems.

Hints: Click below the exercise cells.
Click the Answer button of the palette to summarize your results.
You need a ruler. Measure carefully.

**Questions Open / Close**

Look for the value of $V_0$.
Where can you read $\Delta V$ ? Mark it on your paper. How much is it?

What do the (red) points indicate?
Where can you read $r[V_0 + \Delta V]$? Draw this line on your print out.

Answer : The red points indicate the function value of r[V] for $V_0$.
$r[V_0 + \Delta V]$ is about 108 inches.

Where can you find $r[V_0 + \Delta V] - r[V_0]$? How much is it?
Some guys call this difference $\Delta r$.

Answer : $\Delta r$ is about 10 inches.

Calculate $\Delta r / \Delta V$. This is the average rate of change of the radius with respect to change in volume.

Why can your result differ from the 0.336 in/yd^3 in the plot label?

Answer : $\Delta r / \Delta V = 0.33$ in/yd^3.
Numerical values are more precise than graphical measurements.

What does the slope of the (green) dashed line indicate?

Answer: The slope of the green dashed line indicates the average rate of change of the radius with respect to change in volume of the balloon at $V_0$ during the time interval $\Delta V$.

How many inches did the radius of the balloon increase from starting to $V_0 = 90\ yd^3$?

Answer: The radius increased approximately 110 inches.

Run one movie with $\Delta V$ between **10** $yd^3$ to **20** $yd^3$.
What is the max and min of $\Delta r / \Delta V$?

```
ΔV = ; (* between 10 to 20 yd³ *)
MDBalloonMovie[ΔV]
```

Start ▷ | Stop | Delete × | Print

Print | Close Questions

Close Balloon Movie

## DEFINITION : Difference Quotient

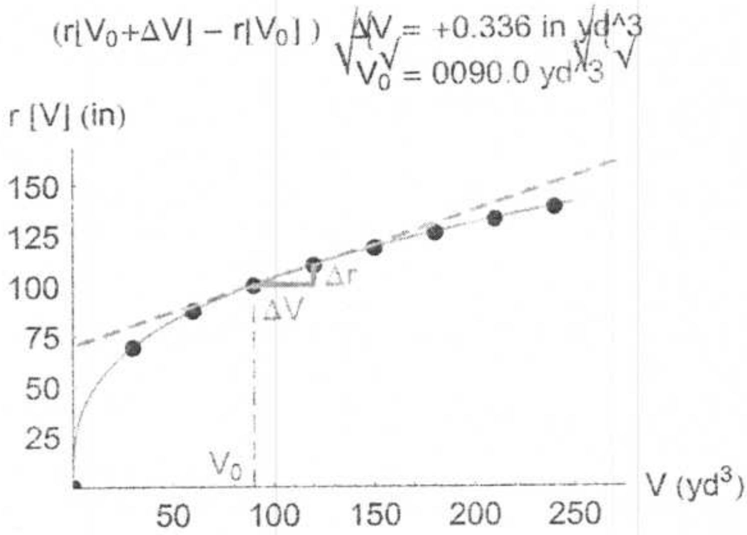
$$\frac{f[a+h]-f[a]}{h}$$

### 2.1 Graphical Explanation

$$\text{Average rate of change} = \frac{\text{Change in f}}{\text{Change in h}} = \frac{f[a+h]-f[a]}{h}$$

$$= \text{Slope of the green dashed secant line}$$

Here you see the change of the radius $\Delta r$ in the balloon example as the volume increases at $\Delta V = 30\ yd^3$.
The average rate of change at $V_0 = 90\ yd^3$ is $0.336\ in / yd^3$.




### 2.2 Definition of the Difference Quotient

A difference quotient is a number measuring the average rate of change of a function f [x] at x = a over a given argument interval h.

$$\frac{f[a+h]-f[a]}{h}$$

Print

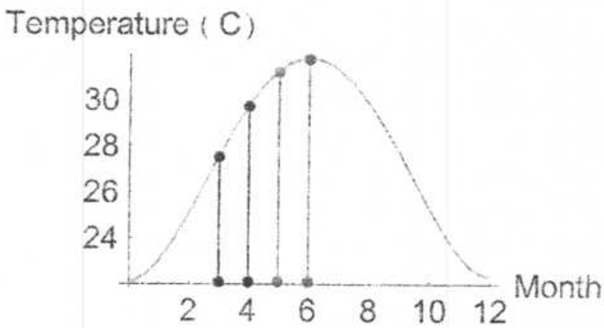
Close Definition

■ **EXAMPLE :**
**Step by Step**

## ■ 3.1 Step by Step

Example: A wonderful brochure of Miami Beach states, that the temperature of the sea in spring rises constantly.
Check this statement for March and May.

Choose h = 1 month.
The function f[x] = 22.1 + 1.069 $x^2$ - 0.178 $x^3$ + 0.00742 $x^4$ models the rising of the temperature.




Solution: Use the difference quotient to calculate the rate of change in degree per month.

Click the Difference Qu button twice.
First for March, a = 3 and then for May, a = 5. See what you get!

```
Clear[f, x];
f[x_] = 22.1 + 1.069 x^2 - 0.178 x^3 + 0.00742 x^4;
```

$$\frac{f[3+1]-f[3]}{1}$$

2.1955

```
Clear[f, x];
f[x_] = 22.1 + 1.069 x^2 - 0.178 x^3 + 0.00742 x^4;
```

$$\frac{f[5+1]-f[5]}{1}$$

0.53982

Now press the Answer button to summarize your result.

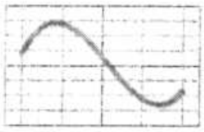
*More details ▼.*

Print

**Close Step by Step**

## EXERCISE SECTION Test Your Knowledge

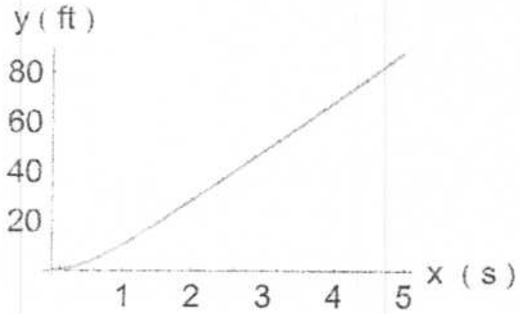
**Practice**



- **Guess it ...**

- **A parachutist jumps ...**

A simple model for a parachutist jump.
Jill jumps from an airplane with a parachute and falls
$f[x] = 12.5\ (e^{-1.6x} - 1) + 20x$ feet in x seconds.




Answer the following questions.

What is the average velocity during the first second?
Solution: 10 ft/s.

Hint: $\Delta x = 1$ s. Click the DifferenceQu button of the palette.

```
Clear[f, x];
f[x_] = 12.5 (e^-1.6x - 1) + 20 x;

f[0 + 1] - f[0]
---------------
       1

10.0237
```

Answer : The average velocity of Jill in the first second is 10 ft/s.

Approximate the velocity at $x_0 = 3$s. Use $\Delta x = 1$ s.
Try this also for $x_0 = 4$s and $x_0 = 5$s.
Solution: 19.9 ft/s. 20ft/s. 20ft/s.

```
Clear[f, x];
f[x_] = 12.5 (e^-1.6x - 1) + 20 x;

f[3 + 1] - f[3]
---------------
       1

19.9179
```



Answer: The average velocity of Jill at $t_0$ = 3s over 1s is 19.9ft/s, at $t_0$ = 4s and at $t_0$ = 5s over 1s is 20ft/s.

Try to describe Jill's parachute jump after $x_0$ = 5 s with respect to the average velocity. Do you find an explanation for this kind of movement?

Hint: Use the results of the former questions. Look at the change of the average velocity and imagine what usually happens when you jump a parachute.

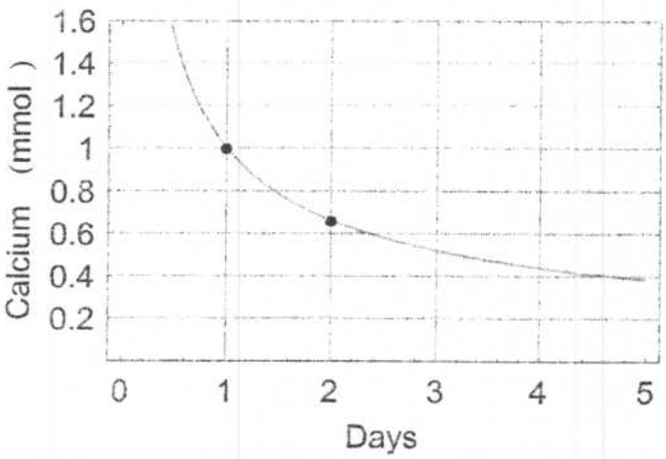
Answer : In the beginning the average velocity increases as long as the parachute is still closed. Then, as soon as the parachute is open, Jill reaches the terminal velocity of about 20ft/s

Print

Close Parachute

- **Removing calcium from the blood**

Chemists found out how fast calcium is removed from the blood of a person. Suppose that t days after an injection of calcium, the amount f of the labeled calcium remaining in the blood can be described by
f[t] = $t^{-0.6}$ for t > 0.5 d. Calcium in mmol.




Calculate the average removing velocity during the first and the second day.
Solution: −0.34mmol per day.

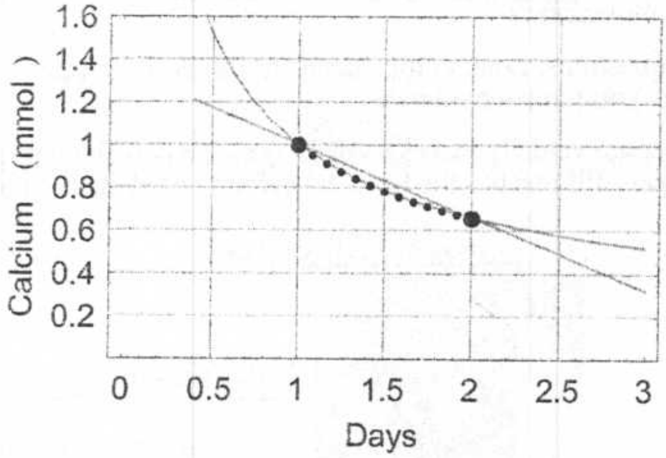
Hint: Click the Difference Qu button and rename BOTH x to t.

```
Clear[f, t];
f[t_] = t^-0.6;

f[1 + 1] - f[1]
---------------
       1

-0.340246
```

Take the second day and calculate the average removing velocity of every second hour. ($\Delta t = 2/24$ h, velocity in mmol / ( 2 h) ).
Start with day 1.

List up your solutions and compare it with the solution of the last question.

One number of your list should be equal or almost equal to the solution of this last question. At which time does this happen?




```
Clear[f, x];
f[x_] = x^-0.6;

Δx = 1 / 12;
start = 1;
stop = 2;

MDShowTable[{"Days \n step 2h", "Ca decrease \n (mmol / 2h)"},
{PaddedForm[x, 3],
    PaddedForm[(f[x + Δx] - f[x]) / Δx, 4]},     {x, start, stop, Δx}]
```

| | |
|---|---|
| 1. | -0.5627 |
| 1.08 | -0.4974 |
| 1.17 | -0.4436 |
| 1.25 | -0.3987 |
| 1.33 | -0.3607 |
| 1.42 | -0.3283 |
| 1.5 | -0.3003 |
| 1.58 | -0.276 |
| 1.67 | -0.2548 |
| 1.75 | -0.2361 |
| 1.83 | -0.2195 |
| 1.92 | -0.2048 |
| 2. | -0.1916 |

Answer :

Print

Close Calcium

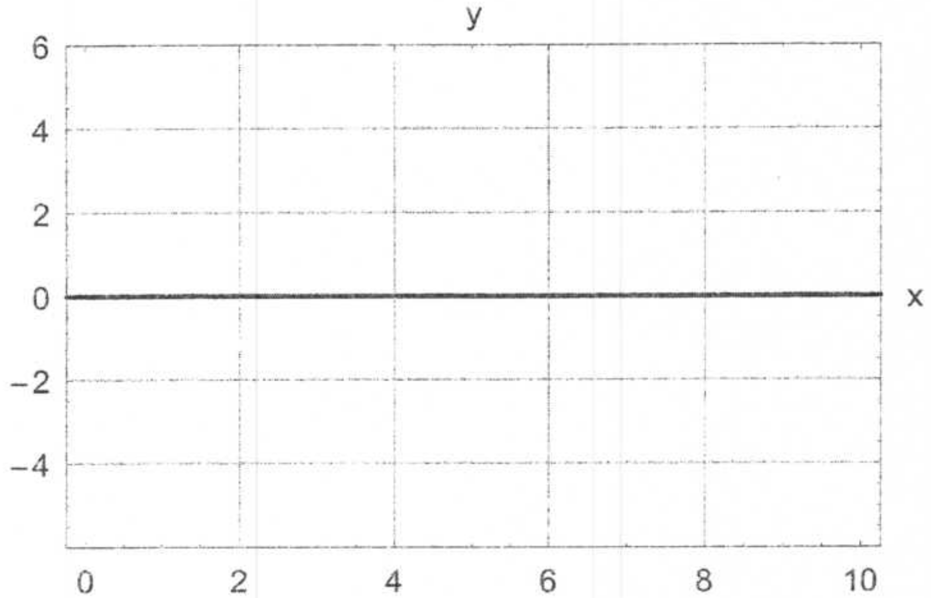
- **Computer Lab**

- **Test Your Knowledge, Keywords**

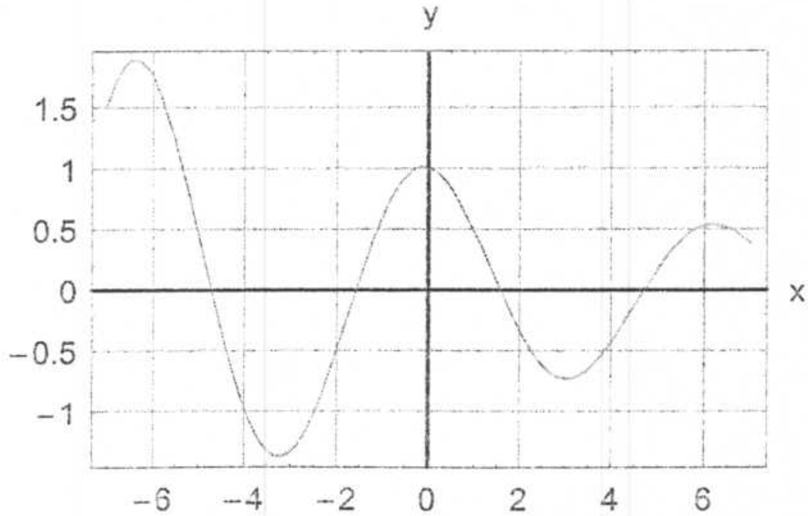
Hint: Print out this section.
Click the Answer button of the palette to write down your solutions.

If h gets smaller, does the difference quotient get smaller as well?

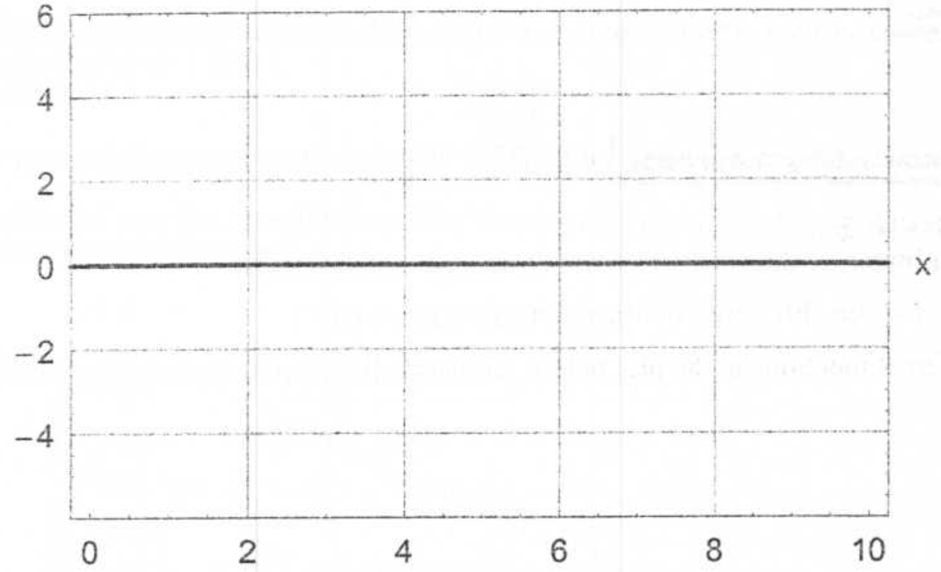
Sketch three different functions in the plot below, so that their difference quotient equals 2 in the interval [3;4] .




Look at the function below. In which of the intervals {−6,−4},{−4,−2}and {−2,2.4} is the difference quotient positive, in which negative and in which 0. Try to answer the question without calculating.




Sketch a function in the interval {1,8}, that is increasing, but not monotonously increasing over the interval {1,4} and decreasing, but not monotonously decreasing over the interval {4,8}.

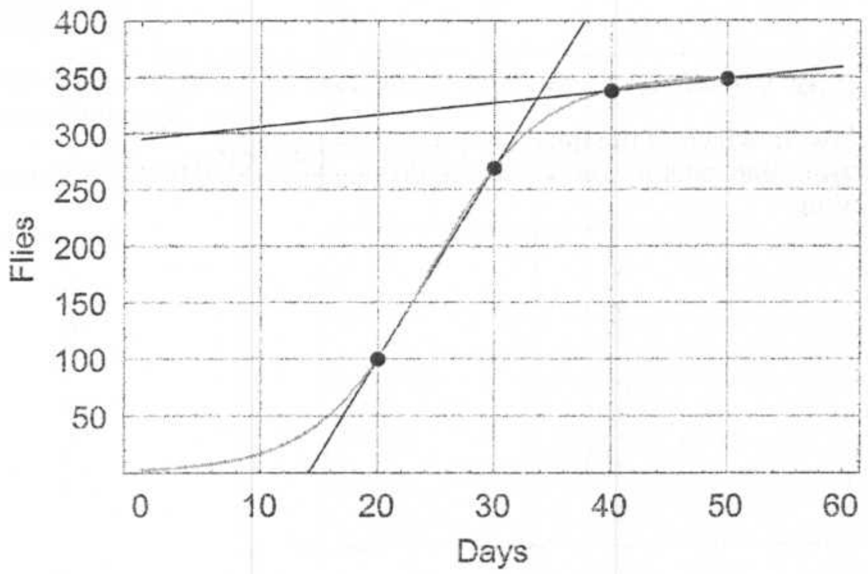



The difference quotient of a function f[x] at a yields 3.27.
Is f at a increasing, approximately 0 or decreasing?

Experimental biologist often want to know the rates at which populations grow under controlled laboratory-conditions.
f[x] models the growth of fruit flies.

What is the average rate of change in the population from day 20 to 30 and from day 40 to 50?
f[x] = $(700 * 69.6^{0.05x}) / (2 * 69.6^{0.05x} + 348)$

Use the Difference Qu button.




Print

Close Test Your Knowledge

**Close Exercise Section**

## SUMMARY & INTERNET Resources